A Proposed Heuristic for a Computer Chess Program

John L. Jerz[1]

Software Engineer, Lockheed Martin

ABSTRACT

How might we create an evaluation function for a computer chess program that plays a stronger positional game of chess? A new heuristic for estimating the positional pressure produced by chess pieces is proposed. We calculate and maintain a database of potential mobility for each chess piece 3 moves into the future, for each position we evaluate in our search tree. We update this piece mobility database dynamically as we evaluate each new chess position (this database allows us to reward chess pieces for specific objectives they can accomplish in the future). We determine the restrictions placed on the future mobility of the pieces based on the mobility of the lower-valued enemy pieces. The central idea is that a better level of insight is obtained by this method. Initial results are presented.

1. INTRODUCTION

This paper is concerned with heuristic algorithms. According to (Koen, 2003) a heuristic is anything that provides a plausible aid or direction in the solution of a problem but is in the final analysis unjustified, incapable of justification, and potentially fallible. Heuristics help solve unsolvable problems or reduce the time needed to find a satisfactory solution.

A new heuristic is proposed which offers better insight on the positional placement of the pieces to a chess-playing computer program. The heuristic will have usefulness in the evaluation function of a computer program, or as part of a teaching tool which explains to a human user the reasons that one side or the other has an advantage in a chess game.

The heuristic involves constructing a table of the future mobility for each piece, taking into account the other pieces on the board, as well as the likely constraints that these other pieces place on this future movement. The heuristic concept is described, and then examples are presented from a software application constructed to demonstrate this concept.

Computer chess programs have historically been weak in understanding concepts relating to positional issues. The proposed heuristic offers a method to potentially play a stronger positional game of chess.

2. PRINCIPLES OF POSITIONAL CHESS

Understanding the principles of positional chess is a necessary starting point before designing concepts useful for a machine implementation. We select the relevant concepts of positional chess which have been addressed by multiple authors.

(Stean, 2002) declares that the most important single feature of a chess position is the activity of the pieces and that the primary constraint on a piece's activity is the pawn structure. (Znosko-Borovsky, 1980) generalizes this principle by declaring that if a piece attacks another, it is not the weaker but the stronger one which has to give way. Stean defines a weak pawn as one which cannot be protected by another pawn, therefore requiring support from its own pieces. This is the ability to be protected by another pawn, not necessarily the existence of such

[1] email:randomvisitor@verizon.net

protection. Stean declares that the pawn structure has a certain capacity for efficiently accommodating pieces and that exceeding that capacity hurts their ability to work together.

(Aagaard, 2003) declares that all positional chess is related to the existence of weakness in either your or the opponent's position. This weakness becomes real when it is possible for the weakness to be attacked. The pieces on the board and their interactions define how attackable these weaknesses are.

(Emms, 2001) declares that is an advantage if a piece is performing several important functions at once, while a disadvantage if a piece is not participating effectively in the game. Emms teaches that doubled pawns can be weak if they are attackable or if they otherwise reduce the mobility of the pawns. Doubled pawns can control vital squares, which might also mean denying mobility to enemy pieces. Isolated pawns require the presence of pieces to defend them if attacked.

(Dvoretsky and Yusupov, 1996) argue that creating multiple threats is a good starting point for forming a plan. Improving the performance of the weakest piece is proposed as a good way to improve your position as a whole.

(McDonald, 2006) gives an example of *good* doubled pawns which operate to restrict the mobility of the opponent's pieces and are not easily attackable. His view is that every position needs to be evaluated according to the unique features present.

(Albus and Meystel, 2001) have written that the key to building practical intelligent systems lies in our ability to focus attention on what is important and to ignore what is not. (Kaplan, 1978) says that it is important to focus attention on the few moves that are relevant and to spend little time on the rest.

(Heisman, 1999) discusses the important elements of positional evaluation, including *global mobility* of the pieces and *flexibility*.

The positional style is distinguished by positional goals and an evaluation which rewards pieces for their future potential to accomplish objectives. (Ulea, 2002) references, then quotes Katsenelinboigen

> the positional player is occupied, first and foremost, with the elaboration of the position that will allow him to develop in the unknown future. In playing the positional style, the player must evaluate relational and material parameters as independent variables. ...The positional style gives the player the opportunity to develop a position until it becomes pregnant with a combination. ...The positional style merely prepares the transformation to a combination when the latter becomes feasible.

(Katsenelinboigen, 1992) further describes the organizational strategy of creating flexible structures and the need to create potential in adaptive systems that face an unpredictable environment. (Botvinnik, 1984) and (Botvinnik, 1970) attempt in general terms to describe a vision for implementing long range planning, noting (Botvinnik, 1970) p.14,22,23:

> An attack must have real features. There must be an object of attack and a path, made up of actual squares, over which the attack is to be carried out... If the attack path is unsafe (closed) the master... looks about him: can he bring other pieces into the game, to better the attack paths (the functions) against the enemy pieces and lessen the attacks against his own pieces? This is what 'positional' play amounts to. I believe that this part of the theory (on positional warfare) is radically different from what has been presented up to now... Play directed at changing a path has the same character, and proceeds by the same formulae, as play for annihilation, with the distinction that now the object of attack is not an enemy piece itself, but a change in the attack-functions.

(Hubbard, 2007) identifies procedures which can be helpful when attempting to measure intangible values, such as the positional pressure produced by chess pieces. (Spitzer, 2007) declares that what gets measured get managed, that everything that should be measured, can be measured, and that we should measure what is most important.

3. GOLDRATT'S THEORY OF CONSTRAINTS AND THINKING PROCESS

(Goldratt and Cox, 2004) have developed a *Theory of Constraints* which postulates that organizations and complex systems are hindered from reaching their goals by the *constraints* placed on that system. Identifying those constraints and removing them can speed progress towards these goals. (Scheinkopf, 1999) describes how Golratt's institute began to modify the original concepts to serve the needs of clients who wanted more generalized procedures to solve a wider variety of problems outside of a factory production environment. (Dettmer, 2007) explores Goldratt's *Thinking Process* and identifies procedures to logically identify and eliminate undesirable effects from systems and organizations. (Dechter, 2003) explains that a model of reality based on constraints helps us to achieve an effective focus for search efforts, and is similar to the heuristic process that humans use to search for effective solutions in complex situations:

> Constraint satisfaction is a simple but powerful idea. If we can pose our world knowledge as a set of local constraints on the values assigned to variables in global solutions to a problem, then when we satisfy those constraints we solve the problem... For an optimist, a constraint specifies the possible; for a pessimist, equivalently, the impossible... These restricted, modular knowledge representations are natural to humans... The lesson is that constraint-based systems are tools that let us make computation more transparent, efficient and reliable - in short, more useful... The power of making the implicit constraints explicit is that they can, in turn, control the search for global solutions, quickly terminating impossible [JLJ - or unlikely] partial solutions or sub-problems and avoiding thrashing search behavior. This can make the search vastly more efficient, sometimes even making apparently intractable problems tractable... One reason these algorithms are so practical is that we can easily tradeoff the degree of consistency, and thus the level of explicitness, with the work required to compute it... A constraint is a restriction on a space of possibilities; it is a piece of knowledge that narrows the scope of this space. Because constraints arise naturally in most areas of human endeavor, they are the most general means for formulating regularities that govern our computational, physical, biological, and social worlds... Although observable in diverse disciplines; they all share one feature in common: they identify the impossible, narrow down the realm of possibilities, and thus permit us to focus more effectively on the possible. Formulating problems in terms of constraints has proven useful for modeling fundamental cognitive activities... as well as having application for engineering tasks... Formulating problems in terms of constraints enables a natural, declarative formulation of *what* must be satisfied, without having to say *how* it should be satisfied.

4. TRANSFORMING PERFORMANCE MEASUREMENT

(Spitzer, 2007) speaks about the critical need to develop metrics which are predictive and which measure strategic potential. We seek to measure how 'ready' our pieces are for supporting strategy, especially when the future positions we face are not entirely determinable.

> Predictive Measures... Today, most measurement still focuses on the past and the present, and it does not serve effectively as a guide for the future. This is because traditional measurement can do nothing except collect data on what has already happened. The winners in business must be able to see beyond the obvious, and be able to manage the future. According to DiPiazza and Eccles, 'Although measurement is inherently based on information about events that have already happened, certain measures can be predictive in nature when [the] relationship among value drivers are well understood.' That is a major reason why measurement frameworks are so important...
> Kaplan and Norton, of Balanced Scorecard fame, have recently started taking leadership in an [area] they call the 'strategic readiness of intangible assets,' and how this readiness can be measured.20 Intangible assets are said to be 'strategically ready' when they can be used to support a strategic objective (like 'increased new products'), which in turn is linked with measures of strategic success (like revenue, profit, or market share). It is not enough just to have intangible assets. The competitive advantage of organizations in the new economy is increasingly dependent on how 'ready' their intangible assets are for deployment in supporting strategy. Intangibles assets that are not ready are like unused inventory. If they cannot be effectively used to support strategic objectives, their value is reduced, sometimes to zero.

5. THE POSITIONAL EVALUATION FUNCTION

We construct an evaluation function with the goal of making our machine more knowledgeable with regard to the positional concepts discussed above. In designing our evaluation function, we heed the advice of (Dombroski, 2000):

> The evaluative criteria are the yardsticks by which the ideas are measured. These evaluative criteria are used to test and verify the strength of your ideas. The criteria are really a further measure of the problem-solver's sensitivity to the problem. The evaluative criteria are used to anticipate all the effects and consequences that can occur in accomplishing a solution to the problem as defined... It is important to choose carefully and select evaluative criteria that will thoroughly test the ideas against reality. The better the evaluative criteria, the better ideas or decisions will evolve from the evaluation phase.

We adopt the vision of (Katsenelinboigen, 1992), pp.62:

> Potential is a measure of the system's predisposition to development. This approach is different from the probabilistic one that deals, albeit in terms of frequencies, with fully objective connections between the present and future states of the system. A potential forms a structure aimed at inducing the events in the environment in which the system is immersed to the system's advantage, preparing the system to channel unexpected outcomes in a way that is favorable to the system, and absorbing or reducing the shocks of unexpected events harmful to the system.

We follow the suggestion in (Pearl, 1984) to use as a strategy an evaluation based on a *relaxed constraint* model, one that ideally provides (like human intuition), a stream of tentative, informative advice for managing the steps that make up a problem-solving process, and use a suggestion in (Fritz, 1989) that structure influences behavior. In order to more accurately estimate the distant positional pressure produced by the chess pieces, as well as to predict the future capability of the pieces in a basic form of planning (Lakein, 1974) (Shoemaker, 2007) we create the software equivalent of a *diagnostic probe* which performs a heuristic estimate of the ability of each piece to accomplish positional objectives. These objectives (as a starting point) will be attacking enemy pieces and supporting friendly pieces (especially those pieces that are weak). We calculate and maintain this database of potential mobility for each chess piece 3 moves into the future, for each position we evaluate.

We update this piece mobility database dynamically as we evaluate each new leaf position in our search tree. This database lets us reward chess pieces for the specific positional objectives they can attack or support in the future (such as 2 moves away from defending a piece or 3 moves away from attacking a square next to the enemy king). Note that the piece *mobility* we calculate is not itself the desired objective - it is the *means* through which we determine the pressure the piece can exert on a distant objective.

We reduce our bonus for each move that it takes the piece to accomplish the desired objective. We then consider *restrictions* which are *likely* to constrain the piece as it attempts to make moves on the board.

For example, let's consider the pieces in the starting position (Figure 1). What squares can our knight on g1 influence in 3 moves, and which squares from this set are likely off-limits due to potential constraints from the enemy pieces? We now introduce the concept of the *influence diagram* (Shoemaker, 2007) and the *engagement diagram* (Figure 2, a new concept), which are interpreted in the following way. If a piece is on our influence diagram for the knight, then it is possible to attack it or defend it in 3 moves (this includes waiting moves or moves which move a piece out of the way). We label this kind of map an *influence diagram* because it accurately shows the squares that the piece can influence in 3 moves, provided that it is unconstrained in movement by the enemy.

Keep in mind that we need to take into account the location of the other pieces on the chessboard when we generate these diagrams for each piece. If we trace mobility through a friendly piece, we must consider whether or not we can move this piece out of the way before we can continue to trace mobility in that particular direction. If we trace mobility through an enemy piece, we must first be able to spend 1 move capturing that piece.

Comparing this 3-move map with a diagram of the starting position, we can determine that the white knight on g1 can potentially attack 3 enemy pieces in 3 moves (black pawns on d7, f7 and h7). We can defend 8 of our own pieces in 3 moves (the knight cannot defend itself).

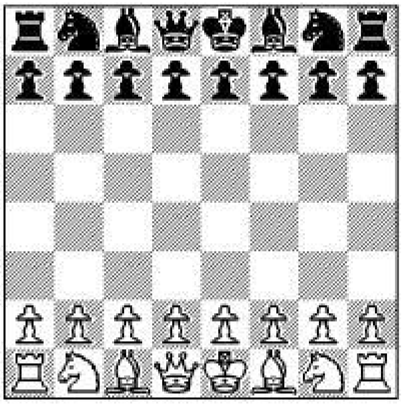

Figure 1: The chess pieces at the starting position

			3		3		3
		3		3		3	
	3		3	2	3	2	3
		3	2	3	2	3	2
	3	3	3		1	3	1
		3	2	1	2	3	2
		3		2		N	

			X		X		X
		X		X		X	
	3		3	2	3	2	3
		3	2	3	2	3	2
	3	3	3		1	3	1
		3	2	1	2	3	2
		3		2		N	

Figure 2: Influence Diagram and Engagement Diagram, Ng1 at starting position

We decide to reward pieces for their ability to accomplish certain types of worthwhile positional objectives: attacking enemy pieces, defending friendly pieces, attacking squares near our opponents king (especially involving collaboration), minimizing our opponents ability to attack squares near our own king, attacking pieces that are not defended or pawns that cannot be defended by neighboring pawns, restricting the mobility of enemy pieces (specifically, their ability to accomplish objectives), etc. In this way, we are getting real about what the piece can do. The bonus we give the piece is 1. a more precise estimate of the piece's ability to accomplish positional objectives that have value and 2. operationally based on real things present on the chessboard. In this way, our positional evaluation function will obtain insight not usually obtained by a computer chess program. It is still an estimate, but the goal here is to focus our search efforts on likely moves in a positional style of play, and to evaluate positions from a more positional point of view.

What does the evaluation function look like for the proposed heuristic? We model (and therefore estimate) the positional pressure of our pieces, by following a two-step process:

1. We determine the *unrestricted future mobility* of each chess piece 3 moves into the future, then

2. We estimate the *operating range* or level of *engagement* of the pieces by determining the limiting factors or constraints that bound the unrestricted mobility.

The concept of using limiting factors is briefly mentioned (Blanchard and Fabrycky, 2006) in the context of Systems Engineering. The consideration of constraints is a part of the *decision protocol of Orasanu and Connolly* (Orasanu and Connolly, 1993) and (Plessner, Betsch and Betsch, 2008) which also includes the identification of resources and goals facing the decision maker. We therefore reduce the bonus for accomplishing objectives (such as, attacking an enemy piece or defending a friendly piece) if the required moves can only be traced through squares that are *likely* to result in the piece being captured before it can accomplish its objective.

We also reduce the engagement bonus for mobility traced through squares where the piece is attacked but not defended.

We may use another scheme (such as probability) for determining mobility reduction for piece movement through squares attacked both by friendly and enemy pieces where we cannot easily resolve whether or not a piece can trace mobility through the square in question.

We reward each piece for its predicted ability to accomplish strategic objectives, exert positional pressure, and restrict the mobility of enemy pieces, based on the current set of pieces on the chess board at the time we are calling our evaluation function. (van Wezel, Jorna and Meystel, 2006) say that anticipation (as a strategy) can be costly and is limited by time constraints. It can hurt our performance if it is not done with competence. An efficient compromise between anticipative and reactive strategies would seem to maximize performance.

We give a piece an offensive score based on the number and type of enemy pieces we can attack in 3 moves. We give a piece a defensive score based on how many of our own pieces it can move to defend in 3 moves. Again, this bonus is reduced for each move it takes to potentially defend such piece. This information is derived from the influence diagram and engagement diagram we just calculated. Extra points can be given for weak or undefended pieces that we can threaten.

The proposed heuristic determines also king safety from these future mobility move maps. We penalize our king if our opponent can move pieces into the 9- square template around our king within a 3 move window. The penalty is larger if the piece can make it there in 1 or 2 moves, or if the piece is a queen or rook. We penalize our king if multiple enemy pieces can attack the same square near our king. Our king is free to move to the center of the board - as long as the enemy cannot mount an attack. The incentive to castle our king will not be a fixed value, such as a quarter pawn for castling, but rather the reduction obtained in the enemy's ability to move pieces near our king (the rook involved in the castling maneuver will likely see increased mobility after castling is performed). The king will come out of hiding naturally when the number of pieces on the board is reduced and the enemy does not have the potential to move these reduced number of pieces near our king. We are likewise free to advance the pawns protecting our king, again as long as the enemy cannot mount an attack on the monarch. The potential ability of our opponent to mount an attack on our king is the heuristic we use as the basis for king safety. Optionally, we will consider realistic restrictions that our own pieces can make to our opponent's ability to move pieces near our king.

Pawns are rewarded based on their chance to reach the last rank, and what they can do (pieces attacked and defended in 3 moves, whether or not they are blocked or movable). The piece mobility tables we generate should help us identify pawns that cannot be defended by other pawns, or other pieces - it is this weakness that we should penalize. Doubled or isolated pawns that cannot be potentially attacked blockaded or constrained by our opponent should not be penalized. Pawns can be awarded a bonus based on the future mobility and offensive/ defensive potential of a queen that would result if it made it to the back rank, and of course this bonus is reduced by each move it would take the pawn to get there.

The information present in the future mobility maps (and the constraints that exist on the board for the movement of these pieces) allow us to better estimate the positional pressure produced by the chess pieces. From these calculations we can make a reasonably accurate estimate of the winning potential of a position, or estimate the presence of positional compensation from a piece sacrifice. This evaluation score also helps steer the search process, as the positional score is also a measure of how interesting the position is and helps us determine the positions we would like to search first.

In summary, we have created a model of positional pressure which can be used in the evaluation function of a computer chess program. (Michalewicz and Fogel, 2004) have this to say about using models to solve problems:

> Whenever you solve a real-world problem, you have to create a model of that problem first. It is critical to make the distinction that the model that you work with isn't the same as the problem. Every model leaves something out. It has to - otherwise it would be as complicated and unwieldy as the real-world itself. We always work with simplifications of how things really are. We have to accept that. Every solution we create is, to be precise, a solution only to the model that we postulate as being a useful representation of some real-world setting that we want to capture.

(Starfield, Smith and Bleloch, 1994) emphasize that problem solving and thinking revolve around the model we have created of the process under study. We can use the proposed model of positional pressure to direct the machine to focus the search efforts on moves with well-placed pieces. For our search efforts, we desire a proper balance between an anticipatory and a reactive planning strategy. Using a suggestion from (van Wezel *et al.*, 2006) we desire our forecast of each piece's abilities to help us anticipate its effectiveness in the game, instead of just reacting to the consequences of the moves.

6. RESULTS

We have created software to implement the proposed heuristic and now examine several positions to see if we can obtain a better positional understanding of how well the pieces are likely performing. John Emms (Emms, 2001),

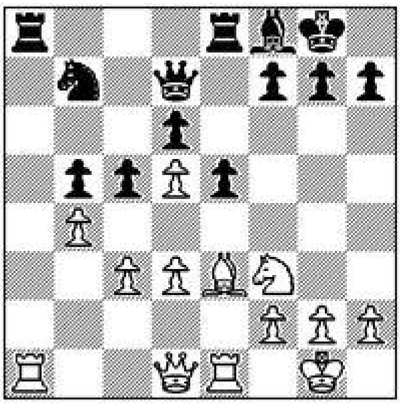

Figure 3: Emms-Miralles (1998), Black to move

reached Figure 3 as white with the idea of restricting the mobility of black's knight on b7. How fully engaged is this piece in the game? Let's see what the *influence diagram* and *engagement diagram* from the proposed heuristic show us: We generate the *constraint maps* as in Figure 4 in order to estimate the squares that the knight

R							
R							
R		P		P			M
P		P		M		M	
R	P	P	P	P	M		M
R	Q		Q	P	P	P	P
R		Q	M	R	M	K	M
Q	R	M	R	M	R	M	K

r	r	r	m	r	r		k
r	q	q		m	q	m	k
r		q	m	p	p	p	p
m	q	p		p	q		
p	p	p	p		p	q	
r							q
r							
r							

Figure 4: Emms-Miralles Constraint maps

on b7 is likely to be denied access.[2] We then apply the constraint maps to the individual vectors which make up the influence diagram to create the engagement diagram. We can see from Figure 5 that the movement of the piece on b7 has been constrained.

Figure 6 examines a sideline from Estrin-Berliner (1965-68 corr.) after the proposed improvement 12.Qe2 Be6 13.Qf2. How fully engaged is the white Bishop on c1? We generate constraint maps and influence diagram as before in order to generate the engagement diagram. We see that the bishop on c1 can enter the game after moving a pawn out of the way.

[2]The simplified version of a *constraint map* assumes that bishop and knight are of equal value, and we use *M* or *m* to represent a white or black *minor* piece.

	3		1		3		
3	n	3	3	3	2	3	
3	3	2	1	2			
1		1		3		3	
3	3	2	3	3	3		
3	2		3	3			
	3		3				
3		3					

	X		1		X		
X	n	X	X	X	2	X	
X	X	X	1	X			
X		X		X		X	
X	X	X	X	X	X		
X	X		X	X			
	X		X				
X		X					

Figure 5: Influence Diagram and Engagement Diagram for Nb7

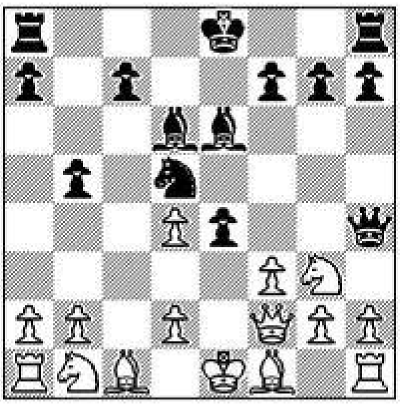

Figure 6: Estrin-Berliner analysis (1965-68 corr.) after 12.Qe2 Be6 13.Qf2, Black to move

			3				
				3		3	
			3		3		2
3		3		3		2	
	3		3		2		3
2		3		2		3	
	1		1		3		
3		B		3			

			X				
				X		X	
			X		X		X
3		3		X		X	
	X		3		X		X
2		3		2		X	
	1		1		3		
3		B		3			

Figure 7: Influence Diagram and Engagement Diagram for Bc1

The computer can use the heuristic knowledge present in the influence diagram and engagement diagram to estimate how *fully engaged* each piece is in the game. The maps are useful in 1) steering our search efforts 2) deciding which positions are the most promising from a positional point of view and 3) predicting the winning potential at the end points of our search tree.

Figure 8 is from Nickel-Hydra (2004 corr.) after 18.g4 was played. The black knight on h6 has been *constrained* to stay on the edge of the board by the white pawns and the location of the other pieces. It is unlikely that Hydra was aware of the long term consequences of the constraints placed on the Black knight on h6. In fact, the knight on h6 never moved the remainder of the game, which white won. The influence diagram and engagement diagram

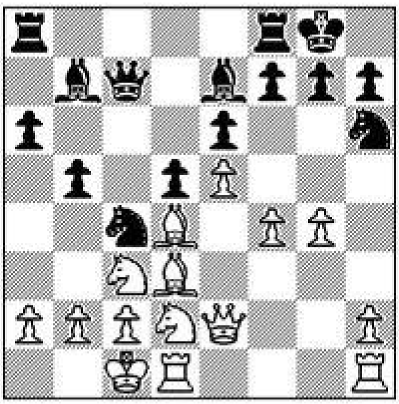

Figure 8: Nickel-Hydra (2004 corr.) Black to move

		3	3	3		1	3
	3		3	2	1	2	3
		3	2	3	2	3	n
	3		3	2	1	3	3
		3	2	3		1	2
	3		3	2	3	2	3
		3		3	2	3	2
			3		3		3

		X	3	X		1	3
	X		X	X	1	X	X
		X	X	X	X	X	n
	X		X	X	X	X	X
		X	X	X		X	X
	X		X	X	X	X	X
		X		X	X	X	X
			X		X		X

Figure 9: Influence Diagram and Engagement Diagram for Nh6

corresponding to the knight on h6 (Figure 9) strongly suggest that this piece has been effectively constrained by correspondence GM Nickel and therefore made ineffective in the game.

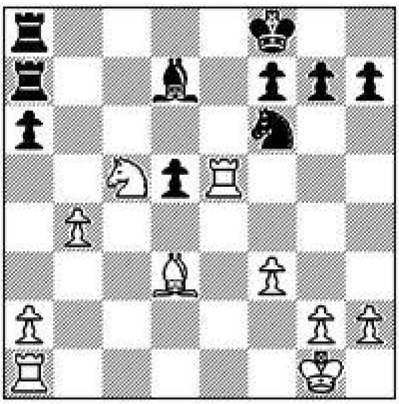

Figure 10: Fischer-Petrosian (1971), white to move

Figure 10 is from Fischer-Petrosian (1971). Fischer was criticized for playing Nxd7 and giving up a well-placed knight for a Bishop which seems to be *bad*. We can use the proposed heuristic to examine the positional placement of the d7 Bishop. The influence diagram and engagement diagram of Figure 11 indicate that the d7 Bishop has influence over much of the board, although it has been constrained generally to a defensive role on the queenside. Perhaps Fischer used a similar insight before deciding to exchange the piece.

2		1		1			
	2		b		2		2
2		1		1		2	
	1		2		1		2
1		2		2		1	
	2		2		2		1
3		2		3		2	
	3		2		3		3

2		1		1			
	2		b		2		X
2		1		1		X	
	1		2		X		X
X		2		X		X	
	X		X		X		X
X		X		X		X	
	X		X		X		X

Figure 11: Influence Diagram and Engagement Diagram for Bd7

7. CONCLUSIONS

The proposed heuristic offers insight on the ability of the chess pieces to accomplish strategic objectives and implements the flexible structures of Katsenelinboigen. It offers promise as a component of an evaluation function for a computer chess program. It also offers promise as a component of a chess tutor, as it can offer insight into the ability of the pieces to accomplish positional objectives. The presented results demonstrate this insight for several test positions. Perhaps chess is more than just calculation (Aagaard, 2004), but the day may come sooner than we think when computers use heuristics to play a positional game of chess at skill levels equal to their current strong tactical play.

Future work will involve construction of a full positional evaluation function as part of a computer chess program.

8. REFERENCES

Aagaard, J. (2003). *Excelling at Positional Chess*. pp.35-36. Glouchester Publishers plc, London.

Aagaard, J. (2004). *Inside the Chess Mind*. pp.157. Glouchester Publishers plc, London.

Albus, J., Meystel, A. (2001) *Engineering of Mind*. pp.80. John Wiley and Sons, New York.

Blanchard, B., Fabrycky, W. (2006). *Systems Engineering and Analysis, Fourth Edition*. pp.162-163. Prentice Hall, Upper Saddle River, New Jersey.

Botvinnik, M. (1970). *Computers, Chess, and Long-Range Planning*. pp.2-3,5,7,9,10,13,14,22,23,24,26,63. Longman Group Ltd., London.

Botvinnik, M. (1984). *Computers in Chess: Solving Inexact Search Problems*. pp.16,38,39,64,67,105-106. Springer-Verlag, New York.

Dechter, R. (2003). *Constraint Processing*. pp.v-vi,xvii. Morgan Kaufmann Publishers, San Francisco, California.

Dettmer, H. (2007). *The Logical Thinking Process*. pp.100,127-138. Quality Press, Milwaukee, WI.

Dombroski, T. (2000). *Creative Problem-Solving*. pp.119. toExcel Press, Lincoln, NE.

Dvoretsky, M., Yusupov, A. (1996). *Positional Play*. pp.111,121. Henry Holt and Company, New York.

Emms, J. (2001). *Simple Chess*. pp.7,63,67,70. Everyman Publishers plc, London.

Fritz, R. (1989). *The Path of Least Resistance: Learning to Become the Creative Force in Your Own Life*. pp.5,6-7,8,9,11,12,31,38-39,49,51-54,116-118,133-137. Fawcett Columbine.

Goldratt, E., Cox, J. (2004). *The Goal*. pp.307. North River Press, Great Barrington, MA.

Heisman, D. (1999). *Elements of Positional Evaluation, Third Edition*. pp.38,40,136. Chess Enterprises, Moon Township, Pennsylvania.

Hubbard, D. (2007). *How to Measure Anything*. pp.23,39,43,115. John Wiley and Sons, Hoboken, New Jersey.

Kahneman, D., Slovic, P., Tversky, A. (1982). *Judgment under Uncertainty: Heuristics and Biases*. pp.69. Cambridge University Press, New York.

Kaplan, J. (1978). *The ABCs of Chess. In: Hochberg, B., Chess Life and Review, July*. pp.370. New Windsor, NY.

Katsenelinboigen, A. (1992). *Indeterministic Economics*. pp.15,50,52-53,62,66,67,70,71,72-73,132,138,252-253. Praeger Publishers, New York.

Koen, B.V. (2003). *Discussion of the Method*. pp.28. Oxford University Press, New York.

Lakein, A. (1974). *How to Get Control of Your Time and Your Life*. pp.25. Signet, New York.

McDonald, N. (2006). *The Art of Planning in Chess*. pp.35,86. Batsford, London.

Michalewicz, Z., Fogel, D. (2004). *How to Solve It: Modern Heuristics*. pp.31. Springer, New York.

Orasanu, J., and Connolly, T. (1993). *The Reinvention of Decision Making. In: Klein, G.A., Orasanu, J., Calderwood, R. and Zsambok, C.E. (eds.) Decision Making in Action: Models and Methods*. pp. 3-20. Ablex, Norwood, New Jersey.

Pearl, J. (1984). *Heuristics*. pp.xi,115,131. Addison-Wesley, Reading, MA.

Plessner, H., Betsch, C., Betsch, T. (2008). *Intuition in Judgment and Decision Making*. pp.120-121. Lawrence Erlbaum Associates, New York.

Scheinkopf, L. (1999). *Thinking for a Change*. pp.2,3,4. St. Lucie Press, Boca Raton, Florida.

Shoemaker, P. (2007). *Forecasting and Scenario Planning: The Challenges of Uncertainty and Complexity. In: Koehler, D., Harvey, N. (eds.) Blackwell Handbook of Judgment and Decision Making*. pp.281,283. Blackwell Publishing, Malden MA.

Spitzer, D. (2007). *Transforming Performance Measurement*. pp.42,68-69,79,84-85,209,226. American Management Association, New York.

Starfield, A., Smith, K., Bleloch, A. (1994). *How to Model It: Problem Solving for the Computer Age*. pp.21. Burgess International Group, New York.

Stean, M. (2002). *Simple Chess*. pp.7,39,42,132. Dover Publications, Mineola, New York.

Ulea, V. (2002). *The Concept of Dramatic Genre and The Comedy of A New Type: Chess, Literature, and Film*. pp.17-18. Southern Illinois University Press, Carbondale and Edwardsville.

van Wezel, W., Jorna, R., Meystel, A. (2006). *Planning in Intelligent Systems*. pp.23,65,96. John Wiley and Sons, Hoboken, New Jersey.

Znosko-Borovsky, E. (1980). *The Middle Game in Chess*. pp.12. Dover Publications, Inc., New York